

## A comparison between the method of lines and Adomian decomposition method for solving the KdV-Burger equation

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### Abstract

This paper presents two methods for obtaining the solutions to the nonlinear Korteweg-de Vries–Burgers (KdVB) equation. The first is the method of lines (MOL). The second method is Adomian decomposition method (ADM). The numerical results of the MOL are compared with the analytical results of the ADM. In order to show the reliability of the considered methods we have compared the obtained solutions with the exact ones. The results reveal that the both methods are effective and convenient for solving such types of partial differential equations but the method of lines gives accurate results over the analytical method.

**Keywords:** KdV-Burger equation; the method of lines; Adomian decomposition method; finite difference scheme; Runge–Kutta method.

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### 1. Introduction

This paper is concerned with the initial-boundary value problem associated with the nonlinear dispersive and dissipative wave which was formulated by Korteweg, de Vries and Burgers in the form

$$\frac{\partial u}{\partial t} + \mu u \frac{\partial u}{\partial x} - \theta \frac{\partial^2 u}{\partial x^2} + \delta \frac{\partial^3 u}{\partial x^3} = 0 \quad (1)$$

Where  $\mu, \theta, \delta$  are constant coefficients.

It is well known that many physical phenomena can be described by the Korteweg-de Vries–Burgers equation. Eq. (1) can serve as a nonlinear wave model of a fluid in an elastic tube [1], of a liquid with small bubbles [2,] and turbulence [3,4]. The coefficients  $\theta$  and  $\delta$  in Eq. (1) represent the damping and the dispersion coefficients, respectively. We note that Eq.(1) is non integrable.

Soliton solutions of the KdV equation are known since long time [5,6]. Many problems, however, involve not only dispersion but also dissipation, and these are not governed by the KdV equation. More complicated problems are the flow of liquids containing gas bubbles [7,8], and the propagation of waves in an elastic tube filled with a viscous fluid [9,10]. Other cases regarded the governing evolution equation can be shown to be the so-called Korteweg-de Vries–Burgers equation.

In particular, the travelling wave solution to the KdVB equation has been studied extensively. Johnson [11], Demiray [12] and Antar and Demiray [13] derived KdVB equation as the governing evolution equation for waves propagating in fluid-filled elastic or viscoelastic tubes in which the effects of dispersion, dissipation and nonlinearity are present.

The KdVB equation is a one-dimension generalization of the model description of the density and velocity fields that takes into account pressure forces as well as the viscosity and the dispersion. It may be a more flexible tool for physicists than the Burgers equation. Several studies in the literature, employing a large variety of methods to derive explicit solutions for KdVB equation (1).

## 2. The Method of Lines

The method of lines [14] is a well established numerical technique (or rather a semi analytical method) for the analysis of transmission lines, waveguide[15-18]. The method of lines is regarded as a special finite difference method but more effective with respect to accuracy and computational time than the regular finite difference method. It basically involves discretising a given differential equation in one or two dimensions while using analytical solution in the remaining direction. The MOL has the merits of both the finite difference method and analytical method, it does not yield spurious modes nor have the problem of relative convergence. The MOL is generally recognized as a comprehensive and powerful approach to the numerical solution of time-dependent partial differential equations (PDEs). This method usually proceeds in two separate steps: first, approximating the spatial derivatives. Second, the resulting system of semi discrete (discrete in space–continuous in time) ordinary differential equations (ODEs) is integrated in time. The essence of the method of lines is a way of approximating PDEs by ODEs. Obviously, an advantage of the MOL is that one can use all kinds of ODE solvers and techniques to solve the semi-discrete ODEs directly.

## 3. Solving the KdV-Burger equation using the MOL

Consider KdV-Burger equation (1) with the initial condition

$$u(x, 0) = \left(\frac{1}{25}\right)\left(\frac{25c}{v} - 100 v^2 \delta + \frac{\theta^2}{\delta}\right) + 12v^2 \delta \operatorname{sech}^2(vx) - \frac{12}{5} v \theta \tanh(x) \quad (2)$$

and the boundary conditions

$$u(a, t) = 0.98 \quad , \quad u(b, t) = 0.02 \quad (3)$$

The exact solution of this problem is given by

$$u(x, t) = \left(\frac{1}{25}\right)\left(\frac{25c}{v} - 100 v^2 \delta + \frac{\theta^2}{\delta}\right) + 12v^2 \delta \operatorname{sech}^2(vx - ct) - \frac{12}{5} v \theta \tanh(vx - ct) \quad (4)$$

The solution domain of the KdV- Burger equation (1) is the rectangle  $a \leq x \leq b$  ,  $0 \leq t \leq T$  Let us subdivide it into uniform rectangular meshes by the lines  $x_i = ih$  ( $i=0,1,2,3,\dots,N$ ) and the lines  $t_j = jk$  ( $j=1,2,3,\dots$ ), We replace the partial derivatives depend on spatial variables  $u_x$ , dissipation term  $u_{xx}$  and dispersion term  $u_{xxx}$  in KdV-Burger equation (1) with known finite difference approximations at point  $x_i$ .

The solution of the method of lines using fourth order finite difference scheme for  $u_x$  ,  $u_{xx}$  , and  $u_{xxx}$  is denoted by MOL1, however the solution using a second order finite difference scheme for  $u_x$  ,  $u_{xx}$  and  $u_{xxx}$  is denoted by MOL2.

The derivative  $u_x$  in KdV-Burger equation (1) is computed by finite differences scheme in two way

1. second order approximations  $u_x = \frac{u_{i+1}-u_{i-1}}{2h} + O(h^2)$
2. fourth order approximations  $u_x = \frac{u_{i-2}-8u_{i-1}+8u_{i+1}-u_{i+2}}{12h} + O(h^4)$

The derivative  $u_{xx}$  in KdV-Burgers equation (1) is computed by finite differences in two ways

1. second order approximations  $u_{xx} = \frac{u_{i-1}-2u_i+u_{i+1}}{h^2} + O(h^2)$
2. fourth order approximations  $u_{xx} = \frac{-u_{i-2}+16u_{i-1}-30u_i+16u_{i+1}-u_{i+2}}{12h^2} + O(h^4)$

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2. fourth order approximations  $u_{xxx} = \frac{u_{i-3}-8u_{i-2}+13u_{i-1}-13u_{i+1}+8u_{i+2}-u_{i+3}}{8h^3} + O(h^4)$

Applying the above finite difference schemes to Eq. (1) yields a system of ordinary differential equations for the unknown  $u_i$  as functions in  $t$  as follows:

$$\frac{du_i(t)}{dt} = f(u_i) \quad , \quad i = 1(1)N - 1 \quad (5)$$

Using the fourth order finite difference scheme for  $u_x$ ,  $u_{xx}$  and  $u_{xxx}$  (MOLI), we have

$$\begin{aligned} \frac{du_i(t)}{dt} = & \delta \frac{(u_{i-3}(t) - 8u_{i-2}(t) + 13u_{i-1}(t) - 13u_{i+1}(t) + 8u_{i+2}(t) - u_{i+3}(t))}{8h^3} \\ & - u_i(t) \frac{u_{i-2}(t) - 8u_{i-1}(t) + 8u_{i+1}(t) - u_{i+2}(t)}{12h} \\ & + \theta \frac{-u_{i-2} + 16u_{i-1} - 30u_i + 16u_{i+1} - u_{i+2}}{12h^2} \quad , i = 1,2,3, \dots \dots N - 1 \end{aligned}$$

And for the second order finite difference scheme both  $u_x$  and  $u_{xxx}$  (MOLII), we have

$$\frac{du_i(t)}{dt} = -\delta \frac{(-u_{i-2}(t)+2u_{i-1}(t)+2u_{i+1}(t)+8u_{i+2}(t))}{2h^3} + \theta \frac{u_{i-1}-2u_i+u_{i+1}}{h^2} - u_i(t) \frac{u_{i+1}(t)-u_{i-1}(t)}{2h} \quad , i = 1,2,3, \dots \dots N - 1$$

Thus, we have the system of differential equations of one independent variable  $t$ . This system can be easily solved by using fourth order Runge–Kutta scheme

$$\begin{aligned} U^{n+1} &= U^n + \frac{\Delta t(K_1+2K_2+2K_3+K_4)}{6} \quad , \quad K_1 = F(U^n) \\ K_2 &= F\left(U^n + \frac{\Delta t}{2}K_1\right) \quad , \quad K_3 = F\left(U^n + \frac{\Delta t}{2}K_2\right) \quad , \quad K_4 = F(U^n + \Delta t K_3) \end{aligned}$$

The computational domain is  $[-20, 20] \times [0, 30]$ . The computational results are listed in Tables 1...5.

The results obtained using the method of lines have been compared with the exact solution as a plots of the solution and the absolute error (AE) profiles of the KdV-Burgers equation where  $\theta$  and  $\delta$  are constants at  $c=0.5$ ,  $\Delta t=10^{-3}$ ,  $v = \frac{\theta}{10\delta}$ ,  $t \in [0,30]$ .

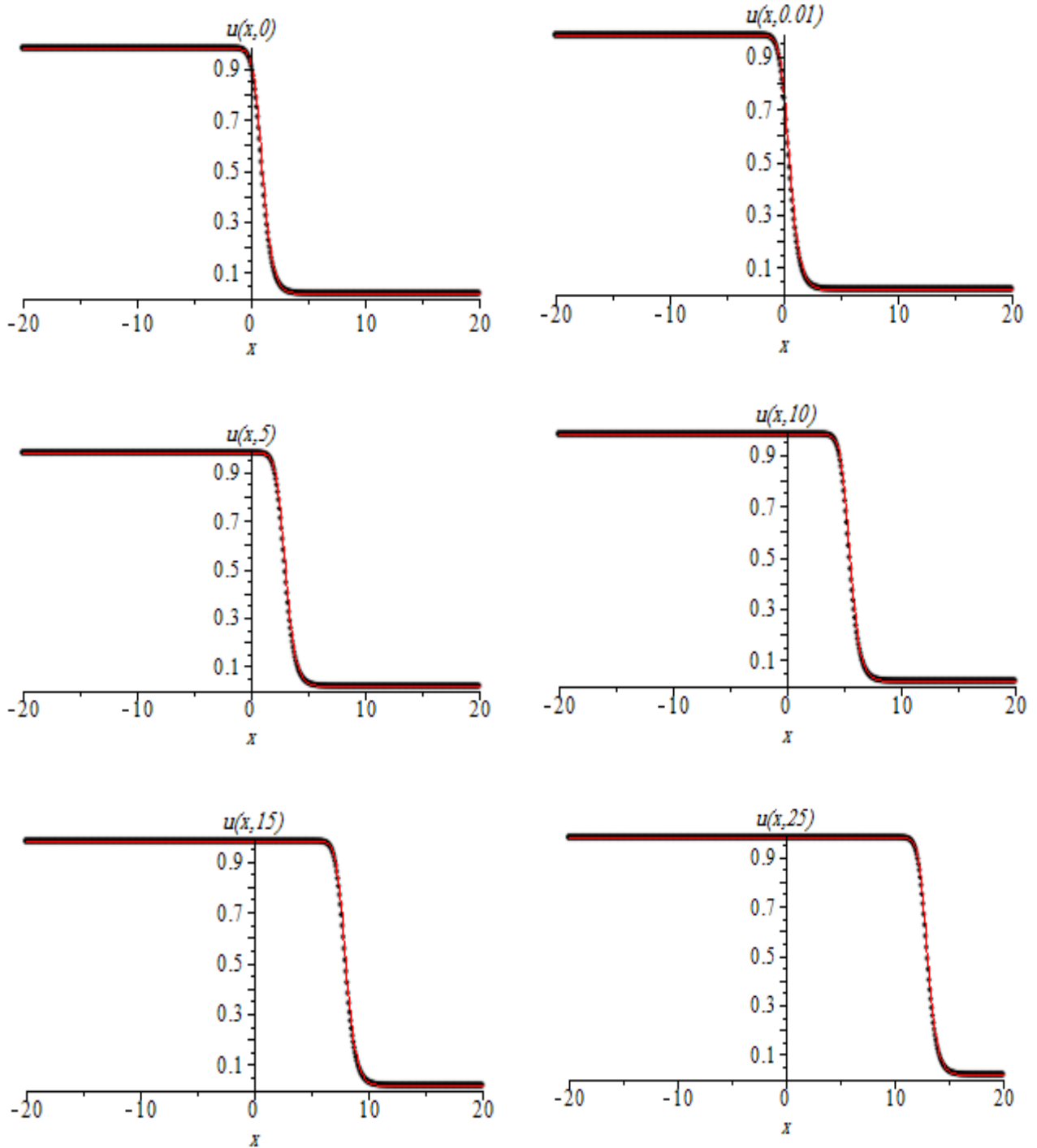


Fig.1. Comparison of MOL I (dotted line) and exact (solid line) solutions  
 at  $N=500$ ,  $\delta = 0.02$ ,  $\theta=0.2$ ,  $c = 0.5$ ,  $v = \frac{\theta}{10\delta}$  and  $t \in [0,30]$ .

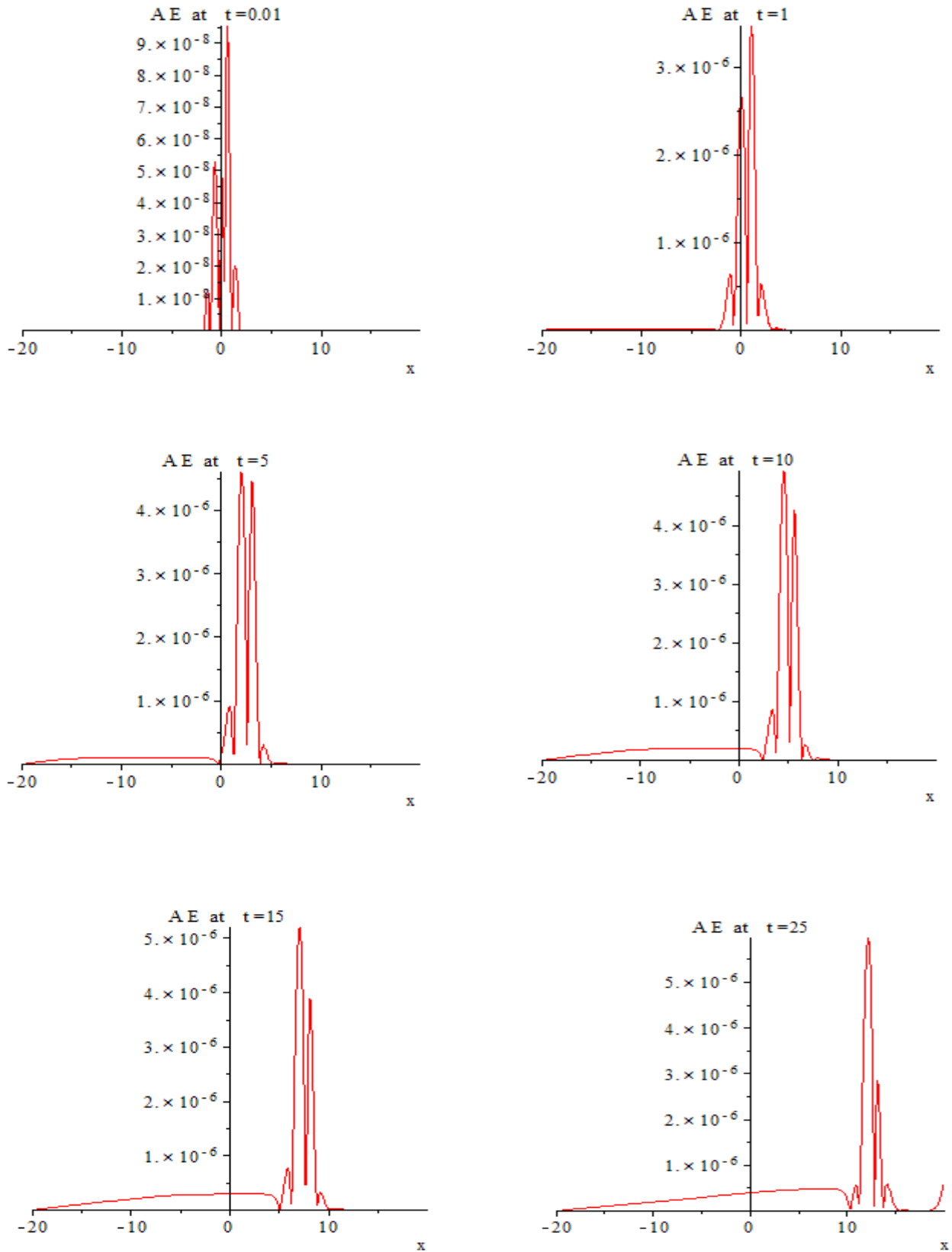


Fig.2. The absolute error between the exact solution  $u(x, t)$  and the (MOL I) solution for KdV-Burger equation for  $t \in [0,30]$ .

We obtain the MOLI solutions of KdV–Burgers equation with higher accuracy than MOLII. The obtained results demonstrate the reliability of the MOL and its wider applicability to nonlinear evolution equations.

#### 4. Adomian decomposition method

Following the analysis of Adomian [Adomian, 1994] equation (1) can be rewritten in an operator form as the following:

$$L(u) + R(u) + N(u) = g(t) \quad (6)$$

where  $L = \frac{\partial}{\partial t}$  is the operator of the highest-ordered derivatives with respect to  $t$  and  $R$  is the remainder of the linear operator. The nonlinear term is represented by  $N(u)$ . Thus we get

$$L(u) = g(t) - R(u) - N(u) \quad (7)$$

The inverse  $L^{-1}$  is assumed an integral operator given by

$$L^{-1} = \int_0^t (\cdot) dt \quad (8)$$

The operating with the operator  $L^{-1}$  on both sides of Eq. (9) we have

$$u = f_0 + L^{-1}(g(t) - R(u) - N(u)) \quad (9)$$

Where  $f_0$  is the solution of homogeneous equation  $L(u) = 0$  (10)

The integration constants involved in the solution of homogeneous equation (10) are to be determined by the initial or boundary condition according as the problem is initial-value problem or boundary - value problem. The ADM assumes that the unknown function  $u(x, t)$  can be expressed by an infinite series of the form

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t) \quad (11)$$

and the nonlinear operator  $F(u)$  can be decomposed by an infinite series of polynomials given by

$$F(u) = \sum_{n=0}^{\infty} A_n \quad (12)$$

Where  $u_n(x, t)$  will be determined recurrently, and  $A_n$  are the so-called polynomials of  $u_0, u_1, u_2, \dots, u_n$  defined by

$$A_n = \frac{1}{n! a^n} [F(\sum_{i=0}^{\infty} \lambda^i)]_{\lambda=0}, \quad n=0,1,2,3,\dots \quad (13)$$

It is now well known in the literature that these polynomials can be constructed for all classes of nonlinearity according to algorithms set by Adomian [19,22].

### 5. Solution of KdV-Burger equation using ADM

In the following section, we discuss the solution of the KdV-Burger equation using ADM. Eq. (1) can be written in an operator form:

$$L u = -\delta u_{xxx} - u u_x + \theta u_{xx} \quad (14)$$

where the differential operator L is  $L = \frac{\partial}{\partial t}$

Applying the inverse operator  $L^{-1}$  on both sides of (14) And using the decomposition series (12) And (13) yield

$$\begin{aligned} \sum_{n=0}^{\infty} u_n(x, t) = & \left(\frac{1}{25}\right) \left(\frac{25c}{v} - 100 v^2 \delta + \frac{\theta^2}{\delta} + v^2 \delta \operatorname{sech}^2(vx) + -\frac{12}{5} v \theta \tanh(x)\right) \\ & + L^{-1} \left(-\left(\sum_{n=0}^{\infty} A_n\right) - \delta \left(\sum_{n=0}^{\infty} u_n\right)_{xxx} + \theta \left(\sum_{n=0}^{\infty} u_n\right)_{xx}\right) \end{aligned}$$

Where  $A_n$  are Adomian polynomials that represent the nonlinear term  $u u_x$  and given by

$$\begin{aligned} A_0 &= u_{0x} u_0 \\ A_1 &= u_{0x} u_1 + u_{1x} u_0 \\ A_2 &= u_{0x} u_2 + u_{1x} u_1 + u_{2x} u_0 \\ A_3 &= u_{0x} u_3 + u_{1x} u_2 + u_{2x} u_1 \\ A_4 &= u_{0x} u_4 + u_{1x} u_3 + u_{2x} u_2 + u_{3x} u_1 + u_{4x} u_0 \end{aligned} \quad (15)$$

Other polynomials can be generated in a like manner. The first few components of  $u_n(x, t)$  follows as

$$\begin{aligned} u_0(x) &= f(x) \\ u_1(x, t) &= L^{-1}(-A_0 + \theta u_{0xx} - \delta u_{0xxx}) \\ u_2(x, t) &= L^{-1}(-A_1 + \theta u_{1xx} - \delta u_{1xxx}) \\ u_3(x, t) &= L^{-1}(-A_2 + \theta u_{2xx} - \delta u_{2xxx}) \end{aligned} \quad (16)$$

The scheme in (16) can easily determine the components  $u_n(x, t)$ ,  $n \geq 0$ . So it is possible to calculate more components in the decomposition series to enhance the approximation. The resulting components using initial condition (2) and  $\delta = 0.02$ ,  $\theta = 0.2$ ,  $c = 0.5$ ,  $v = \frac{\theta}{10\delta}$ .

$$u_0(x, t) = u_{(x,0)} = \left(\frac{1}{25}\right)\left(\frac{25c}{v} - 100v^2\delta + \frac{\theta^2}{\delta} + 12v^2\delta \operatorname{sech}^2(vx) - \frac{12}{5}v\theta \tanh(x)\right)$$

$$u_1(x, t) = L^{-1}(-A_0 + \theta u_{0xx} - \delta u_{0xxx}) = \frac{0.24t(\sinh(x) + \cosh(x))}{\cosh^3(x)}$$

$$u_2(x, t) = L^{-1}(-A_1 + \theta u_{1xx} - \delta u_{1xxx}) = \frac{0.06t^2(2 \cosh(x) \sinh(x) + 2 \cosh^2(x) - 3)}{\cosh^4(x)} \quad (17)$$

$$u_3(x, t) = L^{-1}(-A_2 + \theta u_{2xx} - \delta u_{2xxx}) = \frac{0.02t^3(-6 \sinh(x) + 2 \sinh(x) \cosh^2(x) - 3 \cosh(x) + 2 \cosh^3(x))}{\cosh^5(x)}$$

So, the solution in a series form is given by

$$u(x, t) = 0.5 + 0.24 \operatorname{sech}^2(x) - 0.48 \tanh(x) + \frac{0.24t(\sinh(x) + \cosh(x))}{\cosh^3(x)}$$

$$+ \frac{0.06t^2(2 \cosh(x) \sinh(x) + 2 \cosh^2(x) - 3)}{\cosh^4(x)}$$

$$+ \frac{0.02t^3(-6 \sinh(x) + 2 \sinh(x) \cosh^2(x) - 3 \cosh(x) + 2 \cosh^3(x))}{\cosh^5(x)}$$

We plot the solution and AE profiles of KdV-Burger equation at  $t= 0.01, 1, 2, 2.5$  using ADM.

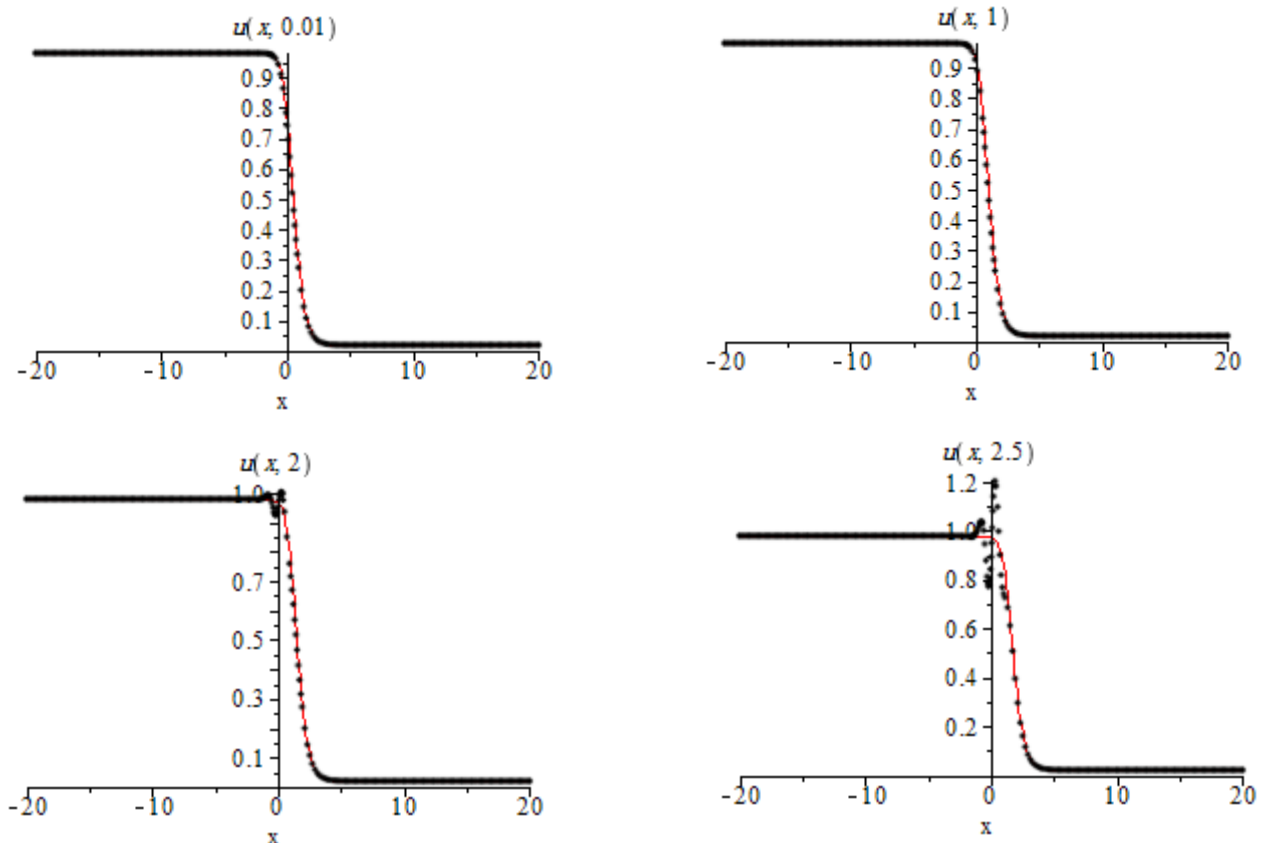


Fig. 3. Comparison of ADM (dotted line) and exact (solid line) solutions corresponding to KdV-Burger equation at  $t=0.01, 1, 2$  and  $2.5$  where  $-20 \leq x \leq 20$ .



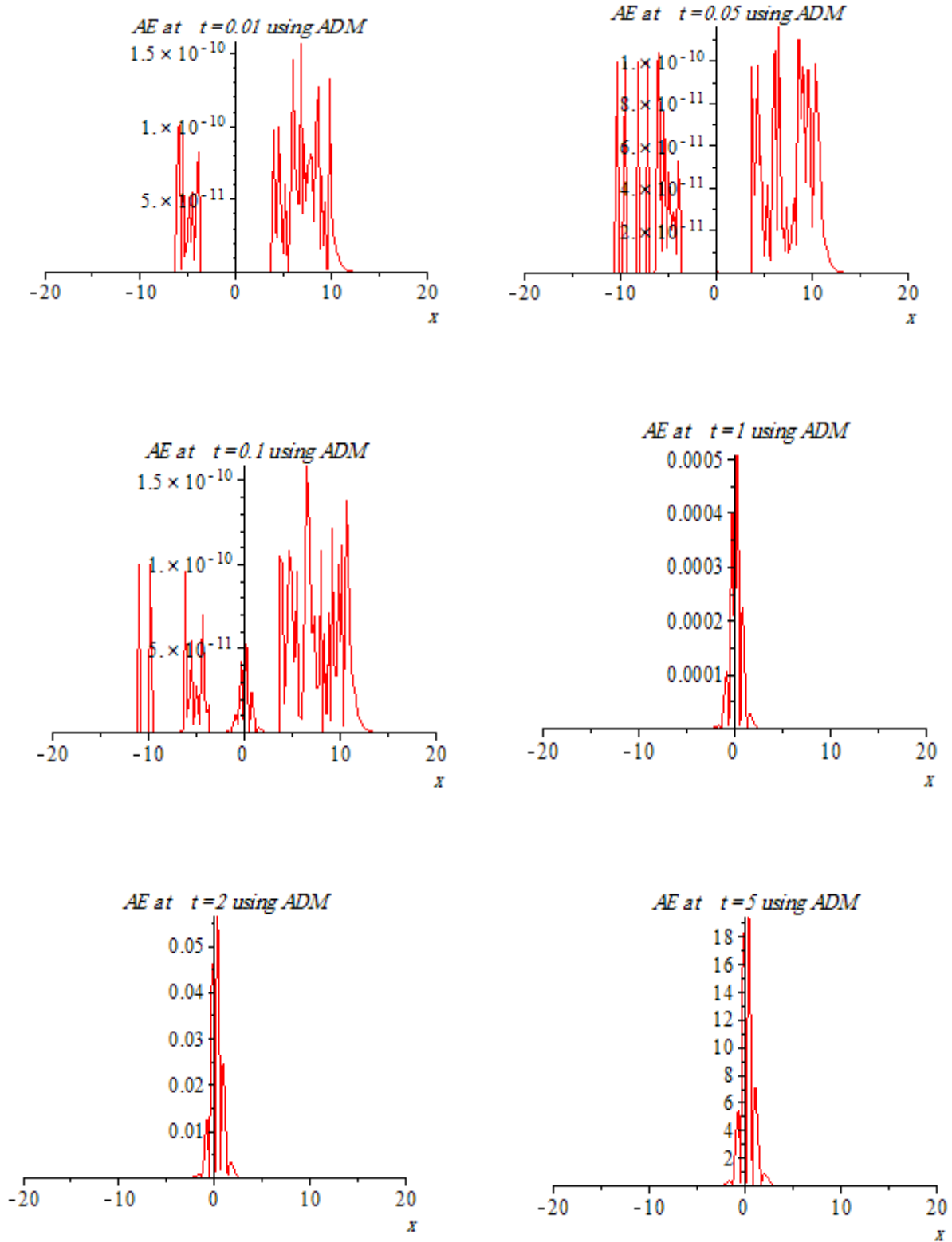


Fig.4. The absolute error between the exact solution  $u(x, t)$  and the (ADM) solution using seven terms for KdV-Burger equation at  $t = 0.01, 0.05, 0.1, 1, 2$  and  $5$  where  $-20 \leq x \leq 20$ .

## 5. Numerical results and some illustrations

In this section, we present the following tables to describe the absolute errors between the exact and numerical solutions. The tables illustrate the errors for both methods, the Adomian decomposition method and the method of lines compared with the exact solution, at different values of  $t$ .

It is observed that if we increase the number of terms in algorithm (17), the size of calculation is maximized with no increase in accuracy so the reduction of terms facilitates the construction of Adomian polynomials for nonlinear operators and gives the same accuracy. ADM can provide the solution with minimal number of iterations.

A comparison between the numerical MOL and the decomposition methods with those obtained by exact solution are given for  $\Delta t=10^{-3}$ . From the tables, we can observe that the decomposition method is accurate as compared with MOL at small period of time but with increasing the time, the MOL is more accurate when compared with ADM.

It is noted that when the time increase by using ADM gradually less accuracy and leads to increased errors. From the comparative study between ADM and the MOL we may conclude that the MOL is more accurate than ADM. To demonstrate the efficiency of our methods we report the absolute errors in some arbitrary points in Tables 1-5.

t=1				
x	Error of MOLI	Error of MOLII	Error of ADM Using 5 terms	Error of ADM Using 7 terms
-6	$1.95076971*10^{-8}$	$1.045180608*10^{-8}$	$1*10^{-10}$	$1*10^{-10}$
-4.8	$1.97223468*10^{-8}$	$1.840800845*10^{-8}$	$5*10^{-10}$	$1*10^{-10}$
-3.6	$1.951909900*10^{-8}$	$5.144083159*10^{-8}$	$3.63*10^{-8}$	$1.05*10^{-8}$
-2.4	$4.6626724703*10^{-8}$	$6.023997785*10^{-8}$	$3.7610*10^{-6}$	$1.0071*10^{-6}$
-1.2	$5.940662235*10^{-7}$	$5.499337451*10^{-4}$	$1.2750*10^{-5}$	$2.91644*10^{-5}$
0	$2.669026577*10^{-7}$	$3.292312710*10^{-5}$	$1.4363*10^{-5}$	$6.85968*10^{-5}$
1.2	$3.148278159*10^{-8}$	$4.142080438*10^{-4}$	$1.9719*10^{-5}$	$3.76501*10^{-5}$
2.4	$3.198081237*10^{-7}$	$2.668147003*10^{-5}$	$5.0092*10^{-5}$	$1.64418*10^{-5}$
3.6	$5.789936818*10^{-9}$	$9.517125206*10^{-6}$	$2.1215*10^{-8}$	$2.7171*10^{-10}$
4.8	$8.49272577*10^{-9}$	$1.006655001*10^{-6}$	$2.0844*10^{-8}$	$2.900*10^{-10}$
6	$3.378234497*10^{-11}$	$9.394625356*10^{-8}$	$1.9060*10^{-8}$	$2.690*10^{-9}$
7.2	$5.128405755*10^{-10}$	$8.426382432*10^{-8}$	$1.750*10^{-8}$	$2.601*10^{-10}$
8.4	$4.189192603*10^{-10}$	$6.594602121*10^{-8}$	$3.30*10^{-8}$	$2*10^{-10}$
9.6	$2.438327352*10^{-10}$	$1.871565402*10^{-8}$	$80*10^{-8}$	$7*10^{-10}$
10.8	$2.263137420*10^{-10}$	$1.420102646*10^{-8}$	$1.30*10^{-8}$	$1.3*10^{-10}$
12	$5.151473414*10^{-10}$	$5.898389068*10^{-9}$	$0.2*10^{-8}$	$2*10^{-10}$
13.2	$3.799035079*10^{-10}$	$5.048499812*10^{-9}$	$1*10^{-8}$	$1*10^{-10}$
14.4	$4.030135947*10^{-10}$	$4.614364446*10^{-9}$	0	0
15.6	$4.006248145*10^{-10}$	$1.144917494*10^{-9}$	0	0

Table 1: The absolute error of MOL and ADM approximation solutions for  $N=500$  and  $t=1$  for KdV-Burger equation.

t= 5				
x	Error of MOLI	Error of MOLII	Error of ADM Using 5 terms	Error of ADM Using 7 terms
-6	9.811727574*10 <sup>-8</sup>	3.184064122*10 <sup>-8</sup>	1.98*10 <sup>-7</sup>	3.05*10 <sup>-7</sup>
-4.8	9.793326538*10 <sup>-8</sup>	3.621059003*10 <sup>-8</sup>	0.0000023854	3.7214*10 <sup>-6</sup>
-2.4	9.791718291*10 <sup>-8</sup>	4.904787680*10 <sup>-8</sup>	0.0305207870	0.734788826
-1.2	9.561010450*10 <sup>-7</sup>	1.087383183*10 <sup>-7</sup>	0.4783218562	2.054157460
0	1.278075613*10 <sup>-8</sup>	1.349134235*10 <sup>-5</sup>	9.460043003	6.074811896
1.2	2.789549492*10 <sup>-7</sup>	4.013708042*10 <sup>-5</sup>	2.405908954	0.513555601
2.4	3.306383080*10 <sup>-6</sup>	1.320023576*10 <sup>-6</sup>	0.4095884839	0.024382254
3.6	1.313720809*10 <sup>-6</sup>	5.262008329*10 <sup>-5</sup>	0.0532855839	4.14292*10 <sup>-4</sup>
4.8	6.730153226*10 <sup>-8</sup>	2.570498745*10 <sup>-5</sup>	0.0071441455	0.000037755
6	2.291066074*10 <sup>-8</sup>	1.209137799*10 <sup>-5</sup>	0.0006701499	0.000003425
7.2	3.507566520*10 <sup>-8</sup>	1.745862402*10 <sup>-6</sup>	0.0000609787	0.004340444
8.4	1.207410716*10 <sup>-9</sup>	1.909985603*10 <sup>-8</sup>	0.0000055334	3.108 * 10 <sup>-8</sup>
9.6	1.932325111*10 <sup>-9</sup>	1.860166550*10 <sup>-8</sup>	5.0169*10 <sup>-7</sup>	2.821*10 <sup>-7</sup>
10.8	1.957402014*10 <sup>-9</sup>	1.685257301*10 <sup>-8</sup>	4.555*10 <sup>-8</sup>	2.57*10 <sup>-8</sup>
12	2.032961226*10 <sup>-9</sup>	1.914493980*10 <sup>-8</sup>	4.14*10 <sup>-8</sup>	2.7*10 <sup>-9</sup>
13.2	1.959853588*10 <sup>-9</sup>	2.581864930*10 <sup>-8</sup>	4.1*10 <sup>-10</sup>	7*10 <sup>-10</sup>
15.6	2.006210853*10 <sup>-9</sup>	6.316530631*10 <sup>-8</sup>	0	0

Table 2: The absolute error of MOL and ADM approximation solutions for  $N=500$  and  $t=5$  for KdV-Burger equation.

t=10				
x	Error of MOLI	Error of MOLII	Error of ADM Using 5 terms	Error of ADM Using 7 terms
-6	1.9560421002*10 <sup>-7</sup>	1.845876784*10 <sup>-7</sup>	7.664*10 <sup>-7</sup>	0.000002454
-4.8	1.9584247256*10 <sup>-7</sup>	2.630796691*10 <sup>-7</sup>	0.000093045	0.000297790
-2.4	1.96117980187*10 <sup>-7</sup>	8.770627557*10 <sup>-7</sup>	1.200017453	40.44833669
-1.2	1.95971405431*10 <sup>-7</sup>	1.5001417885*10 <sup>-7</sup>	24.03445058	41.42833669
0	1.95718343751*10 <sup>-7</sup>	0.00001279501426	276.1600000	40.44833669
1.2	1.92183368330*10 <sup>-7</sup>	0.00036723414692	16.07921173	22.73310721
2.4	5.86544390831*10 <sup>-9</sup>	0.00040572874333	1.105687563	2.955500615
3.6	5.59678794643*10 <sup>-7</sup>	0.00071548482367	0.613696532	0.434259679
4.8	4.3823289428*10 <sup>-8</sup>	2.4049120831*10 <sup>-5</sup>	0.197800572	0.181425669
6	1.9672920730*10 <sup>-8</sup>	1.3278297395*10 <sup>-5</sup>	0.021564028	0.0200777332
7.2	9.11246966527*10 <sup>-7</sup>	2.397605292*10 <sup>-6</sup>	0.001991421	0.0018565813
8.4	3.04337807585*10 <sup>-8</sup>	3.1510888284*10 <sup>-7</sup>	0.000180951	0.0001687194
9.6	3.9962598039*10 <sup>-8</sup>	3.5804745633*10 <sup>-7</sup>	0.000016418	0.0000153083
10.8	2.762818601*10 <sup>-9</sup>	3.6336069673*10 <sup>-7</sup>	0.000001489	0.0000527848
12	3.87726236579*10 <sup>-9</sup>	3.9331884238*10 <sup>-9</sup>	1.3518*10 <sup>-7</sup>	0.0000013888
13.2	3.90540066477*10 <sup>-9</sup>	1.219562827*10 <sup>-9</sup>	1.8941*10 <sup>-8</sup>	1.2605*10 <sup>-9</sup>
15.6	4.0757175020*10 <sup>-9</sup>	6.4857230157*10 <sup>-9</sup>	1.02*10 <sup>-9</sup>	9.401*10 <sup>-9</sup>

Table 3: The absolute error of MOL and ADM approximation solutions for  $N=500$  and  $t=10$  for KdV-Burger equation.

t= 15				
x	Error of MOLI	Error of MOLII	Error of ADM Using 5 terms	Error of ADM Using 7 terms
-6	2.6810669007 *10 <sup>-7</sup>	6.5914829150*10 <sup>-7</sup>	0.000006263	0.0000304262
-4.8	2.7989134165*10 <sup>-7</sup>	1.1667666832*10 <sup>-7</sup>	0.000760262	0.0036916033
-3.6	2.8726718315*10 <sup>-7</sup>	2.1687496243*10 <sup>-7</sup>	0.091381742	0.4412447347
-2.4	2.9121456435*10 <sup>-7</sup>	1.6623724619*10 <sup>-7</sup>	9.835546960	44.47559886
-1.2	2.9299921155*10 <sup>-7</sup>	1.7657941775*10 <sup>-7</sup>	214.2414628	401.2058844
0	2.9380452071*10 <sup>-7</sup>	4.7485260168*10 <sup>-7</sup>	1947.360000	41.85274520
1.2	2.93992226563*10 <sup>-7</sup>	2.1203759859*10 <sup>-7</sup>	887.9344146	3562.564660
2.4	2.93755557767*10 <sup>-7</sup>	1.7390018314*10 <sup>-7</sup>	99.76715580	175.5591839
3.6	2.90562275906*10 <sup>-7</sup>	1.2607281341*10 <sup>-7</sup>	11.74070201	32.81122881
4.8	1.5035556899*10 <sup>-7</sup>	9.633360150*10 <sup>-6</sup>	0.2238971429	2.267795046
6	6.7054763575*10 <sup>-7</sup>	6.0726624081*10 <sup>-5</sup>	0.8501790915	0.663658713
7.2	0.0000050517426	8.3269207999*10 <sup>-4</sup>	0.8296939158	0.8127640893
9.6	1.6956484610*10 <sup>-7</sup>	1.4092512420*10 <sup>-4</sup>	0.0280762015	0.0279368659
10.8	3.2263613634*10 <sup>-8</sup>	2.768658380*10 <sup>-6</sup>	0.0025992930	0.0025866527
12	3.5331507095*10 <sup>-9</sup>	3.0166396709*10 <sup>-4</sup>	0.0002362413	0.0002350946
13.2	3.5331507095*10 <sup>-9</sup>	3.8576692407*10 <sup>-9</sup>	0.0000214347	0.0000213307
15.6	5.7036779084*10 <sup>-9</sup>	5.0145934626*10 <sup>-9</sup>	1.7661*10 <sup>-7</sup>	1.7575*10 <sup>-9</sup>

Table 4: The absolute error of MOL and ADM approximation solutions for  $N=500$  and  $t=15$  for KdV-Burger equation.

t=25				
x	Error of MOLI	Error of MOLII	Error of ADM Using 5 terms	Error of ADM Using 7 terms
-6	2.821226859*10 <sup>-7</sup>	9.39531785*10 <sup>-7</sup>	0.0000856438	0.000700732
-4.8	3.061088577*10 <sup>-7</sup>	9.39531785*10 <sup>-7</sup>	0.0103965637	0.085022365
-3.6	3.300522158*10 <sup>-7</sup>	2.038498259*10 <sup>-7</sup>	1.249903314	10.16613057
-2.4	3.538602846*10 <sup>-7</sup>	2.155697842*10 <sup>-7</sup>	134.8547761	1029.221566
-1.2	3.773140159*10 <sup>-7</sup>	8.775313808*10 <sup>-7</sup>	3142.318964	7149.615382
0	3.999921515*10 <sup>-7</sup>	8.775313800*10 <sup>-7</sup>	23093.26000	4253.757852
1.2	4.213395193*10 <sup>-7</sup>	1.003352956*10 <sup>-7</sup>	11341.68709	68668.29557
2.4	4.403334442*10 <sup>-7</sup>	1.056396081*10 <sup>-7</sup>	1068.966557	2693.452757
3.6	4.560730912*10 <sup>-7</sup>	1.007154359*10 <sup>-7</sup>	138.6426807	590.2571954
4.8	4.685195856*10 <sup>-7</sup>	9.778011733*10 <sup>-7</sup>	12.08368861	55.89151403
6	4.775559218*10 <sup>-7</sup>	9.65407757* 10 <sup>-7</sup>	0.2264402879	4.224219303
8.4	4.840431980*10 <sup>-7</sup>	6.316896550*10 <sup>-7</sup>	0.9502332627	0.917313296
9.6	4.097090497*10 <sup>-7</sup>	4.952353226*10 <sup>-5</sup>	0.9591052401	0.956118795
10.8	5.449300515*10 <sup>-7</sup>	1.892324320*10 <sup>-4</sup>	0.9589183454	0.958647421
12	5.711332285*10 <sup>-6</sup>	8.466932619*10 <sup>-5</sup>	0.8905563998	0.890531822
13.2	2.695173913*10 <sup>-6</sup>	9.816296780*10 <sup>-5</sup>	0.3422403071	0.342238077
15.6	1.959096925*10 <sup>-8</sup>	1.660263860*10 <sup>-5</sup>	0.00388467196	0.003884653

Table 5: The absolute error of MOL and ADM approximation solutions for  $N=500$  and  $t=25$  for KdV-Burger equation.

From the above tables we can infer that ADM have better convergence at small  $t$ . However, a closer look at the errors of ADM reveals that the error considerably increments with increasing the time. This is an indication of little stability on the part of ADM, in contrast to the MOL. By increasing the number of terms not affect on the accuracy of solution.

### **Conclusion**

In this article, the method of lines and Adomian decomposition method have been implemented for obtaining solutions of the KdV-Burger equations. The results show that the considered methods are powerful mathematical tools for obtaining accurate solutions. A comparison between MOL and ADM shows that the accuracy of the MOL is better than that in the ADM for solutions when the time increase. Moreover, MOL can overcome difficulties arising in the calculation of Adomian's polynomials. Therefore the MOL is more convenient to apply than ADM. we conclude that the nonlinear KdV-Burgers equation gives soliton solution, which represents an important application in Physics and physical problems. The computations associated here were performed using Maple 15.

### **References**

- [1] R.S. Johnson, "Nonlinear waves in fluid-filled elastic tubes and related problems", PhD thesis, University of London, London, (1969).
- [2] L.V. Wijngaarden, "One-dimensional flow of liquids containing small gas bubbles", *Philosophical Magazine* 4 ,369–96 (1972).
- [3] D.J. Korteweg, de Vries, " On the Change of Form of Long Waves Advancing in a Rectangular Canal and on a New Type of Long Stationary Waves", *Philosophical Magazine*, 39,422–443 (1895).
- [4] M.J. Ablowitz and P.A. Carkson, *Solitons, " Nonlinear Evolution Equations and Inverse Scatting"*, Cambridge University Press, 200 (1991).
- [5] E. Varley, B.R.Seymour, "A Simple Derivation of the N-Soliton Solutions to the Korteweg - deVries Equation", *Applied Mathematical Sciences*, 58,904-911 (1998).
- [6] W. Hereman, P.P. Banerjee, A. Korpel, "Exact solitary wave solutions of nonlinear evolution and wave equations using a direct algebraic method" , *Journal of Physics*, 19, 607-628 (1986).
- [7] M.A. Helal, "Soliton solution of some nonlinear partial differential equations and its applications in fluid mechanics" , *Applied Mathematics and Computation*,13,1917–29 (2002).
- [8] S.K. Liu," New periodic solutions to a kind of nonlinear wave equations", *Acta Physica Sinica* , 51, 10-14 (2002).
- [9] Mehdi Dehghan," Application of the Adomian decomposition method for two-dimensional parabolic equation subject to nonstandard boundary specifications", *Applied Mathematics and Computation*, 157 ,549–560 (2004).
- [10] A.M. Wazwaz, "The numerical solution of fifth-order boundary value problems by the decomposition method", *Applied Mathematics and Computation* ,136 ,259–270 (2001).
- [11] A.R.Johnson, "nonlinear equation incorporating damping and dispersion", *Journal of Fluid Mechanics*,42,49–60 ( 1970).

- [12] H. Demiray, "Nonlinear waves in a thick walled viscoelastic tube filled with an inviscid fluid", International Journal of Engineering Science, 36,359–62 (1998).
- [13] N. Antar, H. Demiray, "Nonlinear waves in an inviscid fluid contained in a pre-stressed thin viscoelastic tube", Journal of Applied Mathematics and Physics, 48 ,325-340 (1997).
- [14] W.E. Schiesser, "The numerical method of lines: integration of partial differential equations", San Diego, California: Academic Press; (1991).
- [15] W. D. Yang and R. Pregla, "Method of lines for analysis of waveguide structures with multi discontinuities", Electronics Letters ,31 , 892–893 (1995).
- [16] R. Pregla and W. Yang, "Method of lines for analysis of multilayered dielectric waveguides with bragg gratings", Electronics Letters, 29 , 1962–1957 (1993).
- [17] R. Pregla and E. Ahlers, "Method of lines for analysis of discontinuities in optical waveguides", Electronics Letters, October, 29 ,1845-1851 (1993).
- [18] U. Rogge and R. Pregla, "Method of lines for the analysis of strip-loaded optical waveguides", Optical Society of America (B), 8, 459–463 (1991).
- [19] G. Adomian, "A review of the decomposition method in applied mathematics", Journal of Mathematical Analysis and Applications,135 , 501-544 (1998).
- [20] A.M. Wazwaz, "A new algorithm for calculating Adomian polynomial for nonlinear operators", Applied Mathematics and Computation ,17 (111), 53–69 (2000).
- [21] A.M. Wazwaz, "The modified decomposition method and Padé' approximants for solving the Thomas–Fermi equation", Applied Mathematics and Computation, 105 ,11–19 (1999).
- [22] M. M. Mousa and A. Kaltayev, " The Homotopy perturbation Padé' technique for constructing approximate and exact solutions of Boussinesq equations", Applied Mathematical Sciences, 3(22) ,1061-1069 ( 2009).